

AP Statistics  
I-Sample Hypothesis Tests

Name: \_\_\_\_\_

Date: \_\_\_\_\_

- 1) Explain why the statement  $\bar{x} = 50$  is not a legitimate hypothesis.
- 2) For the following pairs, indicate which do not comply with the rules for setting up hypotheses, and explain why:
- a.  $H_0: \mu = 15, H_a: \mu = 15$
  - b.  $H_0: p = .4, H_a: p > .6$
  - c.  $H_0: \mu = 123, H_a: \mu < 123$
  - d.  $H_0: \mu = 123, H_a: \mu = 125$
  - e.  $H_0: \hat{p} = .1, H_a: \hat{p} \neq .1$
- 3) To determine whether the pipe welds in a nuclear power plant meet specifications, a random sample of welds is selected and tests are conducted on each weld in the sample. Weld strength is measured as the force required to break the weld. Suppose that the specifications state that the mean strength of welds should exceed 100lb/in<sup>2</sup>. The inspection team decides to test  $H_0: \mu = 100$  vs.  $H_a: \mu > 100$ . Explain why this hypothesis was chosen rather than  $\mu < 100$ .
- 4) Do state laws that allow private citizens to carry concealed weapons result in a reduced crime rate? The author of a study carried out by the Brookings Institute is reported as saying, "The strongest thing I could say is that I don't see any strong evidence that they are reducing crime." (*San Luis Obispo Tribune*, January 23, 2003).
- a. Is this conclusion consistent with testing:  
 $H_0$ : concealed weapons laws reduce crime vs  $H_a$ : concealed weapons laws do not reduce crime  
or with testing  
 $H_0$ : concealed weapons laws do not reduce crime vs  $H_a$ : concealed weapons laws reduce crime.
- Explain.

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- b. Does the stated conclusion indicate that the null hypothesis was rejected or not rejected? Explain.
- 5) Consider the following quote from the article “Review Finds No Link Between Vaccine and Autism” (*San Luis Obispo Tribune*, October 19, 2005): “We found no evidence that giving MMR causes Crohn’s disease and/or autism in the children that get the MMR.’ Said Tom Jefferson, one of the authors of *The Cochrane Review*. ‘That does not mean it doesn’t cause it. It means we could find no evidence of it.’ “ In the context of a hypothesis test with the null hypothesis being that MMR does not cause autism, explain why the author could not conclude that the MMR vaccine does not cause autism.
- 6) A researcher speculates that because of differences in diet, Japanese children may have a lower mean blood cholesterol level than U.S. children do. Suppose that the mean level for U.S. children is known to be 170. Let  $\mu$  represent the mean blood cholesterol level for all Japanese children. What hypotheses should the researchers test?
- 7) Researchers at the University of Washington and Harvard University analyzed records of breast cancer screening and diagnostic evaluations (“Mammogram Cancer Scares More Frequent than Thought,” *USA Today*, April 16, 1998). Discussing the benefits and downsides of the screening process, the article states that, although the rate of false-positives is higher than previously thought, if radiologists were less aggressive in following up on suspicious tests, the rate of false-positives would fall but the rate of missed cancers would rise. Suppose that such a screening test is used to decide between a null hypothesis is  $H_0$ : no cancer is present and an alternative hypothesis of  $H_a$ : cancer is present. (Although these are not hypotheses about a population characteristic, this exercise illustrates the definitions of Type I and Type II errors.)
- a. Would a false-positive (thinking that cancer is present when in fact it is not) be a Type I error or a Type II error?
- b. Describe a Type I error in the context of this problem, and discuss the consequences of making a Type I error.

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- c. Describe a Type II error in the context of this problem, and discuss the consequences of making a Type II error.
  
- d. What aspect of the relationship between the probability of Type I and Type II errors is being described by the statement in the article that if radiologists were less aggressive in following up on suspicious tests, the rate of false-positives would fall but the rate of missed cancers would rise?

8) Ann Landers, in her advice column of October 24, 1994, (*San Luis Obispo Telegram-Tribune*), described the reliability of DNA paternity testing as follows: “To get a completely accurate result, you would have to be tested, and so would (the man) and your mother. The test is 100% accurate if the man is *not* the father and 99.9% accurate if he is.”

- a. Consider using the results of DNA paternity testing to decide between the following two hypotheses:

$H_0$ : a particular man is the father

$H_a$ : a particular man is not the father

In the context of this problem, describe Type I and Type II errors.

- b. Based on the information given, what are the values of  $\alpha$ , the probability of a Type I error, and  $\beta$ , the probability of a Type II error?
  
  
  
  
  
  
  
  
  
  
- c. Ann Landers also stated, “If the mother is not tested, there is a 0.8% chance of a false positive.” For the hypotheses given in Part (a), what is the value of  $\beta$  if the decision is based on the DNA testing in which the mother is not tested?

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- 9) In a survey conducted by CareerBuilder.com, employers were asked if they had ever sent an employee home because they were dressed inappropriately (June 17, 2008, [www.careerbuilder.com](http://www.careerbuilder.com)). A total of 2765 employers responded to the survey, with 968 saying that they had sent an employee home for inappropriate attire. In a press release, CareerBuilder makes the claim that more than one-third of employers have sent an employee home to change clothes. Do the sample data provide convincing evidence in support of this claim? Test the relevant hypotheses using  $\alpha = 0.05$ .

- 10) The paper “Debt Literacy, Financial Experiences and Over-Indebtedness” (*Social Science Research Network*, Working paper W14808, 2008) included analysis of data from a national sample of 1000 Americans. One question on the survey was:

“You owe \$3000 on your credit card. You pay a minimum payment of \$30 each month. At an Annual Percentage Rate (APR) of 12% (or 1% each month), how many years would it take to eliminate your credit card debt if you made no additional charges?”

Answer options for this question were: (a) less than 5 years; (b) between 5 and 10 years; (c) between 10 and 15 years; (d) never – you will continue to be in debt; (e) don’t know; and (f) prefer not to answer.

- a. Only 354 of 1000 respondents chose the correct answer of never. For the purposes of this exercise, you can assume that the sample is representative of adult Americans. Is there convincing evidence that the proportion of adult Americans who can answer this question correctly is less than 0.40? Use  $\alpha = 0.05$  to test the appropriate hypotheses.

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- b. The paper also reported that 37.8% of those in the sample chose one of the wrong answers (a, b, and c) as their response to this question. Is it reasonable to conclude that more than one-third of adult Americans would select a wrong answer to this question? Use  $\alpha = 0.05$ .

11) The report “2007 Electronic Monitoring & Surveillance Survey: Many Companies Monitoring, Recording, Videotaping – and Firing – Employees” (American Management Association, 2007) summarized the results of a survey of 304 U.S. businesses. Of these companies, 201 indicated that they monitor employees’ web site visits.

- a. Is there sufficient evidence to conclude that more than 60% of U.S. businesses monitor employees’ web site visits? Test the appropriate hypotheses using a significance level of 0.01.

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- b. Is there sufficient evidence to conclude that a majority of U.S. businesses monitor employees' web site visits? Test the appropriate hypotheses using a significance level of 0.01

12) Give as much information as you can about the  $P$ -value of a one-sample  $t$  test in each of the following situations:

- Upper-tailed test,  $df = 8$ ,  $t = 2.0$
- Upper-tailed test,  $n = 14$ ,  $t = 3.2$
- Lower-tailed test,  $df = 10$ ,  $t = -2.4$
- Lower-tailed test,  $n = 22$ ,  $t = -4.2$
- Two-tailed test,  $df = 15$ ,  $t = -1.6$
- Two-tailed test,  $n = 16$ ,  $t = 1.6$
- Two-tailed test,  $n = 16$ ,  $t = 6.3$

13) Paint used to paint lines on roads must reflect enough light to be clearly visible at night. Let  $\mu$  denote the mean reflectometer reading for a new type of paint under consideration. A test of  $H_0: \mu = 20$  versus  $H_a: \mu > 20$  based on a sample of 15 observations gave  $t = 3.2$ . What conclusion is appropriate at each of the following significance levels?

- $\alpha = 0.05$
- $\alpha = 0.01$
- $\alpha = 0.001$

14) The true average diameter of ball bearings of a certain type is supposed to be 0.5 inch. What conclusion is appropriate when testing  $H_0: \mu = 0.5$  versus  $H_a: \mu \neq 0.5$  inch in each of the following situations?

- $n = 13$ ,  $t = 1.6$ ,  $\alpha = .05$
- $n = 13$ ,  $t = -1.6$ ,  $\alpha = 0.5$
- $n = 25$ ,  $t = -2.6$ ,  $\alpha = 0.1$

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- 15) *The Economist* collects data each year on the price of a Big Mac in various countries around the world. The price of a Big Mac for a sample of McDonald's restaurants in Europe in May 2009 resulted in the following Big Mac prices (after conversion to U.S. dollars):

3.80 5.89 4.92 3.88 2.65 5.57 6.39 3.24

The mean price of a Big Mac in the U.S. in May 2009 was \$3.57. Does the sample provide convincing evidence that the mean May 2009 price of a Big Mac in Europe is greater than the reported U.S. price? Test the relevant hypotheses using  $\alpha = 0.05$ .

- 16) Medical research has shown that repeated wrist extension beyond 20 degrees increases the risk of wrist and hand injuries. Each of 24 students at Cornell University used a proposed new computer mouse design, and while using the mouse, each student's wrist extension was recorded. Data consistent with summary values given in the paper "Comparative Study of Two Computer Mouse Designs" (Cornell Human Factors Laboratory Technical Report RP7992) are given. Use these data to test the hypothesis that the mean wrist extension for people using this new mouse design is greater than 20 degrees.

27 28 24 26 27 25 25 24 24 24 25 28  
22 25 24 28 27 26 31 25 28 27 27 25

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17) Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most 150°F, there will be no negative effects on the river's ecosystem. To investigate whether the plant is in compliance with regulations that prohibit a mean discharge water temperature above 150°F, a scientist will take 50 water samples at randomly selected times and will record the water temperatures of each sample. She will then use a z-statistic:

$z = \frac{\bar{x} - 150}{\frac{\sigma}{\sqrt{n}}}$  to decide between the hypotheses  $H_0: \mu = 150$  versus  $H_a: \mu > 150$ , where  $\mu$  is the mean temperature of discharged water. Assume that  $\sigma$  is known to be 10.

- Explain why use of the z-statistic is appropriate in this setting.
- Describe Type I and Type II errors in this context.
- The rejection of  $H_0$  when  $z \geq 1.8$  corresponds to what value of  $\alpha$ ?
- Suppose that the actual value for  $\mu$  is 153 and that  $H_0$  is to be rejected if  $z \geq 1.8$ . draw a sketch of the sampling distribution of  $\bar{x}$ , and shade the region that would represent  $\beta$ , the probability of making a Type II error.
- For the hypotheses and test procedure described, compute the value of  $\beta$  when  $\mu = 153$ .

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f. For the hypotheses and test procedure described, what is the value of  $\beta$  when  $\mu = 160$ ?

g. What would be the conclusion of the test if  $H_0$  is rejected when  $z \geq 1.8$  and  $\bar{x} = 152.4$ ?  
What type of error might have been made in reaching this conclusion?

18) The city council in a large city has become concerned about the trend toward exclusion of renters with children in apartments within the city. The housing coordinator has decided to select a random sample of 125 apartments and determine for each whether children are permitted. Let  $p$  be the proportion of all apartments that prohibit children. If the city council is convinced that  $p > 0.75$ , it will consider appropriate legislation.

a. If 102 of the 125 sampled apartments exclude renters with children, would a level 0.05 test lead you to the conclusion that more than 75% of all apartments exclude children?

b. What is the power of the test when  $p = 0.8$  and  $\alpha = 0.05$ ?

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19) Optical fibers are used in telecommunications to transmit light. Suppose current technology allows production of fibers that transmit light about 50km. Researchers are trying to develop a new type of glass fiber that will increase this distance. In evaluating a new fiber, it is of interest to test  $H_0: \mu = 50$  versus  $H_a: \mu > 50$ , where  $\mu$  denotes the mean transmission distance for the new optical fiber.

a. Assuming  $\sigma = 10$  and  $n = 10$ , find  $\beta$ , the probability of a Type II error, for each of the given alternative values of  $\mu$  when a test with significance level .05 is used:

- i. 52
- ii. 55
- iii. 60
- iv. 70

b. What happens to  $\beta$  in each of the cases in part (a) if  $\sigma$  is actually larger than 10?