## 5.3 Exercises

**1–14** ■ Graph the function.

1. $f(x) = 1 + \cos x$
2. $f(x) = 3 + \sin x$
3. $f(x) = -\sin x$
4. $f(x) = 2 - \cos x$
5. $f(x) = -2 + \sin x$
6. $f(x) = -1 + \cos x$
7. $g(x) = 3 \cos x$
8. $g(x) = 2 \sin x$
9. $g(x) = -\frac{1}{2} \sin x$
10. $g(x) = -\frac{2}{3} \cos x$
11. $g(x) = 3 + 3 \cos x$
12. $g(x) = 4 - 2 \sin x$
13. $h(x) = |\cos x|$
14. $h(x) = |\sin x|$

**15–26** ■ Find the amplitude and period of the function, and sketch its graph.

15. $y = \cos 2x$
16. $y = -\sin 2x$
17. $y = -3 \sin 3x$
18. $y = \frac{1}{2} \cos 4x$
19. $y = 10 \sin \frac{1}{2}x$
20. $y = 5 \cos \frac{1}{4}x$
21. $y = -\frac{1}{3} \cos \frac{1}{3}x$
22. $y = 4 \sin(-2x)$
23. $y = -2 \sin 2\pi x$
24. $y = -3 \sin \pi x$
25. $y = 1 + \frac{1}{2} \cos \pi x$
26. $y = -2 + \cos 4\pi x$

**27–40** ■ Find the amplitude, period, and phase shift of the function, and graph one complete period.

27. $y = \cos\left(x - \frac{\pi}{2}\right)$
28. $y = 2 \sin\left(x - \frac{\pi}{3}\right)$
29. $y = -2 \sin\left(x - \frac{\pi}{6}\right)$
30. $y = 3 \cos\left(x + \frac{\pi}{4}\right)$
31. $y = -4 \sin 2\left(x + \frac{\pi}{2}\right)$
32. $y = \sin \frac{1}{2}\left(x + \frac{\pi}{4}\right)$
33. $y = 5 \cos\left(3x - \frac{\pi}{4}\right)$
34. $y = 2 \sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$
35. $y = \frac{1}{2} - \frac{1}{2} \cos\left(2x - \frac{\pi}{3}\right)$
36. $y = 1 + \cos\left(3x + \frac{\pi}{2}\right)$
37. $y = 3 \cos \pi(x + \frac{1}{2})$
38. $y = 3 + 2 \sin 3(x + 1)$
39. $y = \sin(\pi + 3x)$
40. $y = \cos\left(\frac{\pi}{2} - x\right)$

**41–48** ■ The graph of one complete period of a sine or cosine curve is given.

**(a)** Find the amplitude, period, and phase shift.

**(b)** Write an equation that represents the curve in the form

$$y = a \sin k(x - b) \quad \text{or} \quad y = a \cos k(x - b)$$

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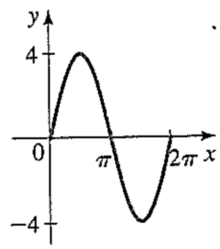


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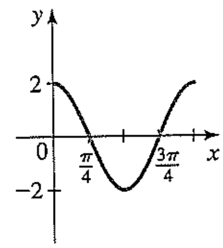


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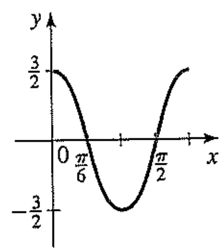


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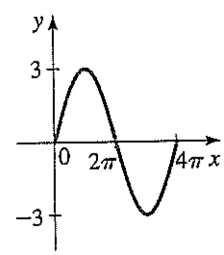


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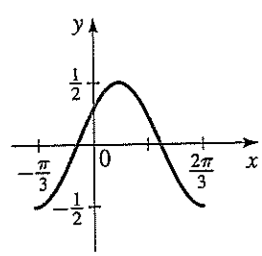


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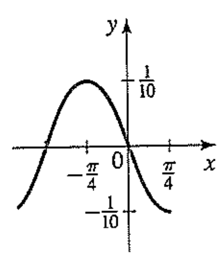


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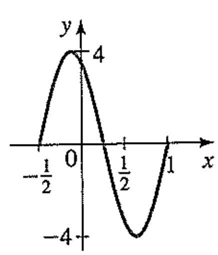


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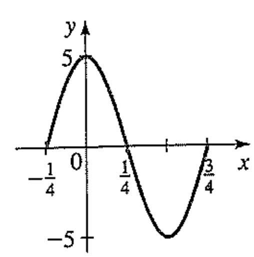



**49–56** ■ Determine an appropriate viewing rectangle for each function, and use it to draw the graph.

49. $f(x) = \cos 100x$
50. $f(x) = 3 \sin 120x$
51. $f(x) = \sin(x/40)$
52. $f(x) = \cos(x/80)$

Taking the natural logarithm of each side gives

$$20c = \ln(10)$$
$$c = \tfrac{1}{20}\ln(10) \approx \tfrac{1}{20}(2.30) \approx 0.12$$

Thus, the damping constant is $c \approx 0.12$. ■

## 5.5 Exercises

**1–8** ■ The given function models the displacement of an object moving in simple harmonic motion.

(a) Find the amplitude, period, and frequency of the motion.

(b) Sketch a graph of the displacement of the object over one complete period.

**1.** $y = 2\sin 3t$

**2.** $y = 3\cos\frac{1}{2}t$

**3.** $y = -\cos 0.3t$

**4.** $y = 2.4\sin 3.6t$

**5.** $y = -0.25\cos\left(1.5t - \dfrac{\pi}{3}\right)$

**6.** $y = -\frac{3}{2}\sin(0.2t + 1.4)$

**7.** $y = 5\cos(\frac{2}{3}t + \frac{3}{4})$

**8.** $y = 1.6\sin(t - 1.8)$

**9–12** ■ Find a function that models the simple harmonic motion having the given properties. Assume that the displacement is zero at time $t = 0$.

**9.** amplitude 10 cm, period 3 s

**10.** amplitude 24 ft, period 2 min

**11.** amplitude 6 in., frequency $5/\pi$ Hz

**12.** amplitude 1.2 m, frequency 0.5 Hz

**13–16** ■ Find a function that models the simple harmonic motion having the given properties. Assume that the displacement is at its maximum at time $t = 0$.

**13.** amplitude 60 ft, period 0.5 min

**14.** amplitude 35 cm, period 8 s

**15.** amplitude 2.4 m, frequency 750 Hz

**16.** amplitude 6.25 in., frequency 60 Hz

**17–24** ■ An initial amplitude $k$, damping constant $c$, and frequency $f$ or period $p$ are given. (Recall that frequency and period are related by the equation $f = 1/p$.)

(a) Find a function that models the damped harmonic motion. Use a function of the form $y = ke^{-ct}\cos\omega t$ in Exercises 17–20, and of the form $y = ke^{-ct}\sin\omega t$ in Exercises 21–24.

(b) Graph the function.

**17.** $k = 2, \quad c = 1.5, \quad f = 3$

**18.** $k = 15, \quad c = 0.25, \quad f = 0.6$

**19.** $k = 100, \quad c = 0.05, \quad p = 4$

**20.** $k = 0.75, \quad c = 3, \quad p = 3\pi$

**21.** $k = 7, \quad c = 10, \quad p = \pi/6$

**22.** $k = 1, \quad c = 1, \quad p = 1$

**23.** $k = 0.3, \quad c = 0.2, \quad f = 20$

**24.** $k = 12, \quad c = 0.01, \quad f = 8$

### Applications

**25. A Bobbing Cork** A cork floating in a lake is bobbing in simple harmonic motion. Its displacement above the bottom of the lake is modeled by

$$y = 0.2\cos 20\pi t + 8$$

where $y$ is measured in meters and $t$ is measured in minutes.

(a) Find the frequency of the motion of the cork.

(b) Sketch a graph of $y$.

(c) Find the maximum displacement of the cork above the lake bottom.

**26. FM Radio Signals** The carrier wave for an FM radio signal is modeled by the function

$$y = a\sin(2\pi(9.15 \times 10^7)t)$$

where $t$ is measured in seconds. Find the period and frequency of the carrier wave.

**27. Predator Population Model** In a predator/prey model (see page 432), the predator population is modeled by the function

$$y = 900\cos 2t + 8000$$

where $t$ is measured in years.

(a) What is the maximum population?

(b) Find the length of time between successive periods of maximum population.

**28. Blood Pressure** Each time your heart beats, your blood pressure increases, then decreases as the heart rests between beats. A certain person's blood pressure is modeled by the function

$$p(t) = 115 + 25\sin(160\pi t)$$

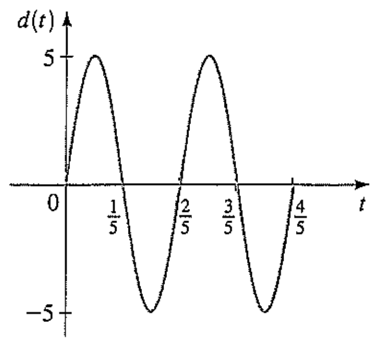
where $p(t)$ is the pressure in mmHg at time $t$, measured in minutes.

(a) Find the amplitude, period, and frequency of $p$.

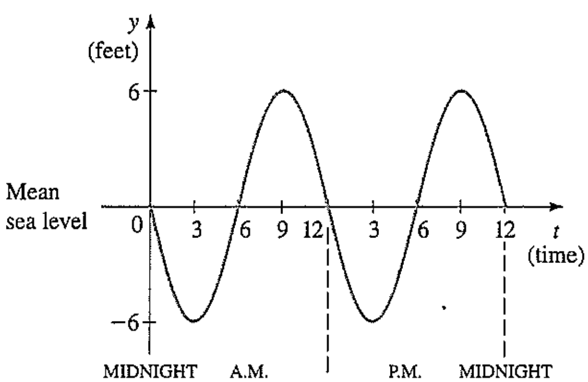
(b) Sketch a graph of $p$.

(c) If a person is exercising, his heart beats faster. How does this affect the period and frequency of $p$?

29. **Spring–Mass System** A mass attached to a spring is moving up and down in simple harmonic motion. The graph gives its displacement $d(t)$ from equilibrium at time $t$. Express the function $d$ in the form $d(t) = a \sin \omega t$.

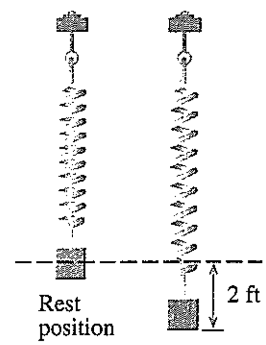



30. **Tides**. The graph shows the variation of the water level relative to mean sea level in Commencement Bay at Tacoma, Washington, for a particular 24-hour period. Assuming that this variation is modeled by simple harmonic motion, find an equation of the form $y = a \sin \omega t$ that describes the variation in water level as a function of the number of hours after midnight.




31. **Tides** The Bay of Fundy in Nova Scotia has the highest tides in the world. In one 12-hour period the water starts at mean sea level, rises to 21 ft above, drops to 21 ft below, then returns to mean sea level. Assuming that the motion of the tides is simple harmonic, find an equation that describes the height of the tide in the Bay of Fundy above mean sea level. Sketch a graph that shows the level of the tides over a 12-hour period.

32. **Spring–Mass System** A mass suspended from a spring is pulled down a distance of 2 ft from its rest position, as shown in the figure. The mass is released at time $t = 0$ and allowed to oscillate. If the mass returns to this position after 1 s, find an equation that describes its motion.




33. **Spring–Mass System** A mass is suspended on a spring. The spring is compressed so that the mass is located 5 cm above its rest position. The mass is released at time $t = 0$ and allowed to oscillate. It is observed that the mass reaches its lowest point $\frac{1}{2}$ s after it is released. Find an equation that describes the motion of the mass.

34. **Spring–Mass System** The frequency of oscillation of an object suspended on a spring depends on the stiffness $k$ of the spring (called the *spring constant*) and the mass $m$ of the object. If the spring is compressed a distance $a$ and then allowed to oscillate, its displacement is given by

$$f(t) = a \cos \sqrt{k/m}\, t$$

(a) A 10-g mass is suspended from a spring with stiffness $k = 3$. If the spring is compressed a distance 5 cm and then released, find the equation that describes the oscillation of the spring.

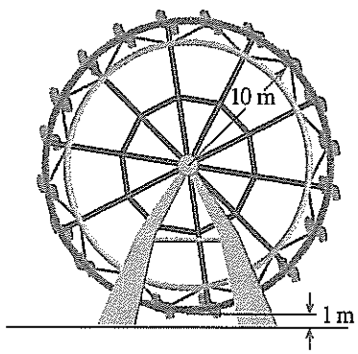
(b) Find a general formula for the frequency (in terms of $k$ and $m$).

(c) How is the frequency affected if the mass is increased? Is the oscillation faster or slower?

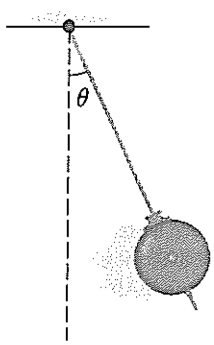
(d) How is the frequency affected if a stiffer spring is used (larger $k$)? Is the oscillation faster or slower?

35. **Ferris Wheel** A ferris wheel has a radius of 10 m, and the bottom of the wheel passes 1 m above the ground. If the ferris wheel makes one complete revolution every 20 s, find an

equation that gives the height above the ground of a person on the ferris wheel as a function of time.




**36. Clock Pendulum** The pendulum in a grandfather clock makes one complete swing every 2 s. The maximum angle that the pendulum makes with respect to its rest position is 10°. We know from physical principles that the angle $\theta$ between the pendulum and its rest position changes in simple harmonic fashion. Find an equation that describes the size of the angle $\theta$ as a function of time. (Take $t = 0$ to be a time when the pendulum is vertical.)




**37. Variable Stars** The variable star Zeta Gemini has a period of 10 days. The average brightness of the star is 3.8 magnitudes, and the maximum variation from the average is 0.2 magnitude. Assuming that the variation in brightness is simple harmonic, find an equation that gives the brightness of the star as a function of time.

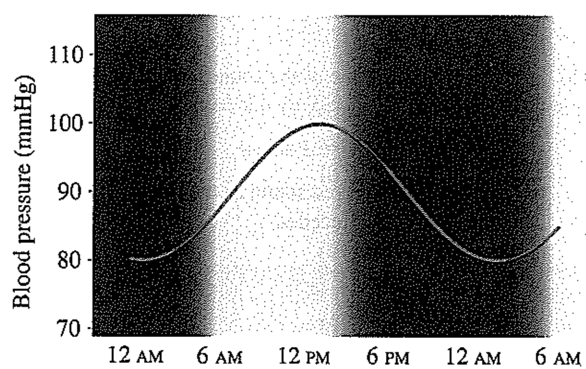
**38. Variable Stars** Astronomers believe that the radius of a variable star increases and decreases with the brightness of the star. The variable star Delta Cephei (Example 4) has an average radius of 20 million miles and changes by a maximum of 1.5 million miles from this average during a single pulsation. Find an equation that describes the radius of this star as a function of time.

**39. Electric Generator** The armature in an electric generator is rotating at the rate of 100 revolutions per second (rps). If the maximum voltage produced is 310 V, find an equation that describes this variation in voltage. What is the rms voltage? (See Example 6 and the margin note adjacent to it.)

**40. Biological Clocks** *Circadian rhythms* are biological processes that oscillate with a period of approximately 24 hours. That is, a circadian rhythm is an internal daily biological clock. Blood pressure appears to follow such a rhythm. For a certain individual the average resting blood pressure varies from a maximum of 100 mmHg at 2:00 P.M. to a minimum of 80 mmHg at 2:00 A.M. Find a sine function of the form

$$f(t) = a\sin(\omega(t - c)) + b$$

that models the blood pressure at time $t$, measured in hours from midnight.




**41. Electric Generator** The graph shows an oscilloscope reading of the variation in voltage of an AC current produced by a simple generator.

(a) Find the maximum voltage produced.

(b) Find the frequency (cycles per second) of the generator.

(c) How many revolutions per second does the armature in the generator make?

(d) Find a formula that describes the variation in voltage as a function of time.